Impact of Market Design and Trading Network Structure on Market Efficiency

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Abstract. This paper investigates the influence of market design, market size, and trading network structure on market efficiency and trade participation rate. The study considers two market designs: Zero Intelligence Traders (ZIT) in Chamberlin's bilateral haggling market and a greedy matching of traders on a network. Sellers and buyers are embedded in a random bipartite graph with varying network densities, and markets vary in size from 20 to 2000 traders.
Simulations reveal that greedy matching generally leads to more efficient allocations than ZIT trading networks. By increasing the average degree of a trading network from 1 to 5 or 10, market efficiency can be significantly improved for both market designs, achieving 89% and 95% of maximum efficiency, respectively. The study also contradicts the common belief that larger markets are better, as no significant impact of market size was found. We discuss the policy implications of these results.

Keywords: Zero-Intelligence Traders · bilateral exchange · market efficiency · Hungarian Algorithm · greedy matching algorithm · network science · agent-based simulation (ABM) · bipartite graph

1 Research Objective and Paper Structure

The primary objective of this research paper is to evaluate the collective impact of market design, market size, and trading network structure on market efficiency and trade participation rate. The research problems are identified in Section 2. The paper embarks on a comparative analysis of two distinct market designs described in detail in Section 3: the Zero Intelligence Traders (ZIT) model in Chamberlin's bilateral haggling market defined and characterized in Section 3.1 and a model based on the greedy matching of traders on a random bipartite graph discussed in Section 3.2. This preliminary investigation aims to shed light on complex interplay between all three above factors of market design, market size and trading network structure, and their implications for market performance and trader engagement presented respectively in Sections 4.1 and 4.2. Recommendations for policy makers can be found in Section 4.1. The final conclusions, along with the limitations of the research and directions for future research, are discussed in Section 5.

2 Definition of Research Problem and its Motivation

The objective of this section is to delineate the research gaps and formulate pertinent questions, derived from the literature review presented in this section and the subsequent Section 3. These questions will be partially addressed in Section 4.

Studies on bilateral trading, typically employing experimental or simulation methods, have demonstrated a varied degree of market efficiency dependent on market designs. These range from approximately 70% of market efficiency for Chamberlin's Bilateral Haggling Market, as shown in the experimental studies of [2,3], to 90% or more of market efficiency for the double auction market mechanism. This high efficiency was demonstrated experimentally in [20,21] and through simulation methods in [8]. Both mechanisms operate under the assumption that traders can interact with everyone else. Although in practice, traders may only interact with a few other traders, the theoretical possibility of universal interaction exists. In real-world scenarios, we typically do not have access to everyone and must limit ourselves to a group of friends or acquainted traders. This observation underscores the importance of social networks in trading, as discussed in [10,6].

Given the observation that most of social interactions happen between a limited group of friends or acquainted traders, see [9,10], suggests that a natural extension of models proposed in the literature would be to embed traders in a social network and assess how this feature of real-world phenomena would impact key metrics of such markets, i.e. market efficiency and trade participation. To achieve this objective, authors of this contribution propose the following research agenda of traders on networks that is depicted in Figure 1.

Figure 1 summaries what has been already established in the literature and what is still missing, i.e. constitutes a research gap, and what potential direction of further research can be. This agenda is intended to investigate two drivers of market efficiency:

- market design, i.e. how specific market mechanisms, e.g. Chamberlin's higgling market [2,3], greedy matching, Hungarian algorithm [5,6,19], perfect competition [16,22], impact the market efficiency and trade participation,
- social network structure, i.e. how network structure, e.g. its density, size, clustering [6,10,17], impact key market outcomes.

Two directions of intervention can be investigated separately or as a whole, but still one would like to decompose the final effect into the effect of market design and the effect of network structure.

Researched thoroughly Partially researched (gap) Not researched (gap)		**Bi-partie graph stuctures of buyers and sellers**		
		Complete bipartite graph, i.e. everybody can trade with everybody	Δ = **Impact of network** on efficiency	**Sparse bi-partie graphs** of buyers and sellers
Market design	**Intelligence** Social planner maximizing social wefare, i.e. consumer surplus + profits	**Classical equilibrium price and volume** in demand & supply model, i.e. matching only so called intramarginal buyers and seller, instead of extramarg.	Impact of network for intelligent market design	**Optimal market-clearing prices** solution for graphs, see Damage, et al. (1986), aka **Hungarian Algorithm** → lack of analytical results for efficiency loss
	Δ = **Impact of market design** and **intelligence**	Impact of market design for full graph	Combined Impact	Impact of market design for sparse graph
	Zero-intelligence Traders (Chamberlin's Bilateral Haggling ZIT market, DA), i.e. bottom-up self-organizaing market design	• Simulation and experimental results for **Double Auction**, see Gode, Sunder (1993) • Only experimental, but no simulation or analytical results for **Chamberlin's ZIT**	Impact of network	**Obtaining analytical results** for tractable bi-partie graph structure or demand & supply functions supplemented by simulation results for less tractable cases

Fig. 1: Potential research agenda of traders on networks
Source: Own work

The market design dimension can be thought as a sequence of market designs starting from the most demanding in terms of information load and intelligence power of social planner like perfect competition design or Hungarian Algorithm [5,6,19] through less intelligent such as greedy matching defined in 8 in Section 3, to more emergent and bottom-up designs not requiring a social planner like in Chamberlin's higgling market [2,3].

The network structure dimension should be thought as network changes of two natures: (1) binary type, i.e. weather possible interactions form the complete graph or not, (2) continuous type, i.e. change in other network characteristics, e.g. size, density, clustering.

The existing literature is primarily focusing on scenarios that do not involve networks, which, to be more precise, is equivalent to the complete graph scenario, for most basic economics model, e.g. perfect competition model [16,22]. For Chamberlin's higgling market, there are established experiment result, see [3], and very preliminary simulation results in [2] presented in no systematic way. Establishing a simulation or even analytical properties of Chamberlin's higgling market is an identified research gap, denoted by the point 1 in Figure 1, although this point can be, of course, more broadly comprehended. Moreover, embedding Chamberlin's market traders in a social network and assessing network impact is another research gap, denoted by point 3 in Figure 1. Changing Chamberlin's market design on network into more intelligent market design of greedy matching on network and assessing its impact on the market is another research gap, denoted by point 2 in Figure 1. All those three research gaps will be addressed, to some limited extent, in the Section 4 of simulated results. More thorough analysis including analytical solutions, other market designs, e.g. Double Auction [8,20,21] and more realistic network structures is the matter of future research.

3 Market Designs on Complete and Sparse Bipartite Graphs

The aim of this section is to delineate three distinct market designs that will be scrutinized in Section 4. These include the benchmark perfect competition model, Chamberlin's higgling market (detailed in Section 3.1), and the greedy matching approach (elucidated in Section 3.2). Furthermore, fundamental concepts pertinent to market design, such as social welfare, market efficiency, and bipartite graphs, will be systematically introduced in Subsections 3.1 and 3.2.

3.1 Chamberlin's higgling market vs. perfect competition model

This subsection defines Chamberlin's higgling market design [3], introduces concepts of social welfare, market efficiency, and the ZIT model for use in Section 4. It also evaluates this design using literature on human subject experiments and simulations in comparison to the perfect competition model.

Economics experiments on human subjects. Unlike natural sciences, economics and social sciences are ill-suited for laboratory experiments on economic systems due to prohibitive costs [7]. Economists often resort to abstract and mathematical models for experimentation [12,15]. While macroeconomic experiments are prohibitive, microeconomic experiments have a track record [4,11].

Chamberlin in [3] discusses an experiment where students are divided into sellers (S) and buyers (B). Assume that we have $n \in \mathbf{N}$ agents of each type. Each seller i had a good to sell at a minimum price S_i (this value is sellers *cost*), and each buyer j wanted to buy a good at a maximum price B_j (this value is buyers *willingness to pay*). To simplify further derivations, assume that all values of S_i and B_j are unique. Students negotiated transaction prices p_{ij} such that $S_i \leq p_{ij} \leq B_j$. This ensured non-negative profits (π_i) and consumer surplus (CS_j), as well as non-negative value added from a transaction (i, j) (SW, we also use the term *social welfare* to refer to this value):

$$\pi_i = p_{ij} - S_i \geq 0,$$

$$CS_j = B_j - p_{ij} \geq 0,$$

$$SW(\{(i, j)\}) = \pi_i + CS_j = B_j - S_i \geq 0.$$

Additionally, if multiple transactions take place on the market, then they are denoted as a set T of (i, j) tuples. The social welfare from all transactions in T is defined as follows:

Definition 1. *We define social welfare as:*

$$\boldsymbol{SW}(T) = \sum_{t \in T} SW(\{t\})$$

and scaled social welfare as an average over n agents of each type:

$$\boldsymbol{SSW}(T) = \boldsymbol{SW}(T)/n.$$

Having defined B_j and S_i, we can specify a demand function as $x(p) = |\{j : B_j \geq p\}|$, i.e. the number of buyers willing to buy a good at price p, as well as a supply function $y(p) = |\{i : S_i \leq p\}|$, i.e. the number of sellers willing to sell at price p. Clearly, $x(p)$ is a non-increasing function of p whereas $y(p)$ is non-decreasing. The perfect competition model defines the equilibrium price set $P^* = \{p : x(p) = y(p)\}$. Note that this set is always non-empty as we assumed that all S_i and B_j are distinct. To simplify notation, without affecting further results, we pick one element $p^* \in P^*$ and call it an equilibrium price. The equilibrium volume is defined as $x^* = x(p^*) = y(p^*)$ [22]. Chamberlin constructed demand and supply functions from B_j and S_i respectively and calculated the equilibrium price and volume. Chamberlin's experiments showed that students "traded too much", with actual sales volume and price diverging from equilibrium predictions. This was due to the engagement of extramarginal traders in transactions.

Definition 2 (Intramarginal and extramarginal traders). *The set of intramarginal sellers is defined as $\{i : S_i \leq p^*\}$, while the set of intramarginal buyers is defined as $\{j : B_j \geq p^*\}$. Traders not meeting these conditions are extramarginal.*

While increased trading volume may seem beneficial, the goal for market designers should be to maximize the total social welfare $\mathbf{SW}(T)$ from all trades T that took place on the market. Market efficiency is the ratio of the achieved social welfare to the maximum social welfare:

Definition 3 (Market efficiency).

$$\boldsymbol{Eff}(T) = \frac{\boldsymbol{SW}(T)}{\max_{T'} \boldsymbol{SW}(T')},$$

where the maximum in the denominator is taken over all possible trade sets T' that could take place on the market.

Markets with efficiency lower than 100% are called inefficient.

A few relevant properties are presented below to show the relationship between social welfare, market efficiency, prices, and volume. The first observation is that social welfare is independent of prices since social welfare is driven by the pairs of sellers and buyers engaged in transactions, but not particular transaction prices. Note that the transaction price does not enter the formula for social welfare. Next, a well-known economic result is that the equilibrium price denoted as p^* implies maximum social welfare, see [22]. However, price equilibrium is not a necessary condition for the maximum social welfare, as the maximum social welfare is achieved if and only if all intramarginal traders are trading and nobody else. Still, equilibrium volume is a necessary, although not sufficient, condition for maximum social welfare. Since according of trading among intramarginal pairs of sellers and buyers is a necessary condition for maximum social welfare, so is the number of intramarginal pairs of sellers and buyers, which is fixed and equal to the equilibrium volume. As a direct consequence of the above property, one can conclude that trading more than the equilibrium volume destroys social welfare since the equilibrium volume is the necessary condition for the maximum social welfare.

Chamberlain's results [3] showed that excess transactions involving extra-marginal trades result in efficiency loss. This inefficiency, due to "too much trading", will be further explored in the next subsection.

Simulation of Zero-Intelligence Traders. The above results of loss efficiency had been already reached by Chamberlin in 1933 in his seminal book of *The Theory of Monopolistic Competition* [2], when he was among the first economists to do the following following simulation manually:

Definition 4 (ZIT's Chamberlin bilateral higgling simulation).
Initiate T - the set of trades made so far - to $\emptyset$.
Repeat:

1. *sample the random pair (i^*, j^*) of: (a) a seller $i^* \in \{i : \forall_j (i,j) \notin T\}$ and (b) a buyer $j^* \in \{j : \forall_i (i,j) \notin T\}$ such that neither of them has traded,*
2. *make the pair (i^*, j^*) trade, if $S_{i^*} \leq B_{j^*}$,*
3. *if the pair (i^*, j^*) traded, than update the set: $T \leftarrow T \cup \{(i^*, j^*)\}$*

until no further trade is possible, i.e. $\min_{i:\forall_j (i,j)\notin T} S_i > \max_{j:\forall_i (i,j)\notin T} B_j$.

In this paper, the decision-making behaviour described in Definition 4, in which there is no bargaining and a transaction takes place as soon as it satisfies a necessary condition of $S_i \leq B_j$, is called *Zero-Intelligence Traders (ZIT)*. The name was coined in [8], in which ZIT simulation was employed to evaluate another market design called Double Auction. Experimental evaluation of this market design is reported in [20].

In our study, we will conduct an actual computer simulation for which the above simulation defined in 4 is a special case, since our general model will assume a non-complete bipartite graph of traders. For simulation purposes we will assume that both consumers' willingness to pay and sellers' costs are following identical independent uniform distributions, i.e. S_i, B_j are distributed as iid random variables taken from $U(0,1)$. For this specification, it is easy to show that the maximum social welfare as number of traders $n \to \infty$ is $n/4(1 + o(1))$ with trades taking part for $p^* = 1/2 + o(1)$.

The maximum social welfare of $n/4$ (after ignoring the error term of $o(n)$) will serve as the benchmark for the evaluation of market designs and market structures considered in this and next sections. The market efficiency will measure the social welfare of a particular market design and structure in the relation to the value of maximum welfare.

Below we demonstrate that the market design of Chamberlin's bilateral higgling, as specified in Definition 4, is inefficient, as its market efficiency for identical uniform distributions of willingness to pay and costs as well as number of traders $n \to \infty$ is approximately 73.6%, i.e. approximately 26.4% of value is not achieved.

To show this, we use the following heuristic argument. Let us define the function $X : R^+ \times [0,1] \to [0,1]$ by the following differential equation:

$$\frac{\partial}{\partial t} X(t,v) = -X(t,v) \int_0^{1-v} X(t,z)\,dz,$$
$$X(0,v) = 1 \quad for \quad v \in [0,1].$$

$X(t,v)$ denotes the probability that at time t, a seller (or a buyer) with value v (or $1-v$, respectively) has not yet traded. At the beginning of the process, i.e. $t = 0$, all traders ($v \in [0,1]$) have not yet traded ($X(0,v) = 1$). Instantaneous decrease of this probability, i.e. $\frac{\partial}{\partial t} X(t,v)$, is proportional to the amount of sellers who have not yet traded, i.e. $X(t,v)$, and the amount of buyers who have not yet traded and would be willing to do so, i.e. $\int_0^{1-v} X(t,z)\,dz$.

By monotonicity, it is clear that $\bar{X}(v) = \lim_{t\to\infty} X(t,v)$ exists. $\bar{X}(v)$ gives the limiting probability (as $n \to \infty$) that a seller with value v does not trade, which is also the probability that a buyer with value $1-v$ does not trade. The total fraction of participants who trade is given by:

$$TP = 1 - \int_0^1 \bar{X}(v)\,dv.$$

$\int_0^1 \bar{X}(v)\,dv$ denotes the average non-participation fraction (averaged over all traders with v distributed uniformly), so TP is the average market participation. The total scaled social welfare **SSW** is:

$$\begin{aligned}\mathbf{SSW} &= \int_0^1 v(1-\bar{X}(1-v))dv - \int_0^1 v(1-\bar{X}(v))dv \\ &= \int_0^1 (1-v)(1-\bar{X}(v))dv - \int_0^1 v(1-\bar{X}(v))dv \\ &= \int_0^1 (1-2v)(1-\bar{X}(v))dv.\end{aligned} \tag{1}$$

The last expression denotes the integral of the difference between inverse demand and supply functions, i.e. $(1-v) - v = 1 - 2v$, weighted by the participation rate $(1 - \bar{X}(v))$.

It seems that $\bar{X}(v) = 0$ for $v \leq \frac{1}{2}$, and $\bar{X}(v) \geq 2v - 1$ for $v \geq \frac{1}{2}$ (the actual function has some curvature on the interval $[1/2, 1]$). The intuition is that all intramarginal traders, both sellers and buyers, as defined in Definition 2, do engage in trading, i.e. $\bar{X}(v) = 0$ for $v \leq \frac{1}{2}$. However, we see that also some extramarginal traders do trade, i.e. $\bar{X}(v) \geq 2v - 1$ for $v \geq \frac{1}{2}$. From these bounds we get that $\mathbf{SSW} \geq \int_0^1 \max(2v - 1, 0)^2 \, dv = 1/6$, whereas the maximum SW is 1/4.

According to a numerical calculation, it turns out that $\bar{T} \approx 0.71$ (71% of participants trade), and $\mathbf{SSW} \approx 0.184$, which is approximately 73.6% of the maximum social welfare, which constitutes the market efficiency. Conducted simulations for $n = 1{,}000$ in Section 4 confirm this asymptotic argument.

As mentioned above, the trade participation, i.e. fraction of traders engaged in a transaction, is approximately 71%, instead of 50% as it would be in the efficient situation. This 71% can be decomposed among:

- 50% are intramarginal traders,
- remaining $\approx$ 21% are extramarginal traders, i.e. sellers $\{i : S_i > \frac{1}{2}\}$ and buyers $\{j : B_j < \frac{1}{2}\}$ resulting in "too much trading" and inefficiency.

As a further potential research question would it be interesting to generalize above two propositions and specify conditions regarding the class of demand and supply functions, for which those qualitative and quantitative properties hold.

3.2 Greedy matching of traders on network

The aim of this subsection is to introduce another market design for a bilateral exchange, i.e. greedy matching of traders, in order to compare it to Chamberlin's higgling market from the previous subsection, Subsection 3.1. The performance of greedy matching algorithm is reported in Section 4.

Greedy matching of traders on a complete graph. Given a significant efficiency loss in Chamberlin's higgling market design discussed in Subsection 3.1, there is a need to come up with another market design, which would result in an efficiency gain and make it closer to perfect competition model's outcomes. Such mechanism would require so called social planner who would know traders' evaluations, i.e. B_j and S_i and would match traders according to a specified algorithm, resulting in a higher social welfare than in Chamberlin's higgling market. One such algorithm is defined below:

Definition 5 (Greedy algorithm for traders' matching).

1. *sort sellers in non-decreasing order of* S_i*,*
2. *sort buyers in non-increasing order of* B_j*,*
3. *match greedily until* $B_j < S_i$*.*

This is an efficient algorithm of a linear complexity with regard to the number of traders resulting in the maximum social welfare, provided that each pair of traders (i, j) can trade with each other.

Greedy matching of traders on a non-complete bipartite graph. An interesting extension of the algorithm in Definition 5 is to imagine that traders are embedded onto a random (in particular, non-complete) bipartite graph, see Definition 7. As a result, they cannot trade with everybody, as it would be the case for the complete bipartite graph, see Definition 6, but only with those traders with whom they have a connection to, i.e. there is an edge between them.

Definition 6 (Complete bipartite graph). *Complete bipartite graph* $G = (V, E_{complete})$ *is the pair of:*

1. *the set of vertices* $V = B \cup S$ *such that* $B \cap S = \emptyset$*, i.e. the union of non-overlapping sets of buyers and sellers,*
2. *the set of all possible connections (edges) between sellers and buyers , i.e.* $E_{complete} = \{(i, j) : j \in B, i \in S\} = S \times B$.

Definition 7 (Random bipartite graph with probability p). *Random bipartite graph with probability* $p \in [0, 1]$ *is defined as* $G = (V, E_p)$*, i.e. the pair of:*

1. *the set of vertices* $V = B \cup S$ *such that* $B \cap S = \emptyset$*, i.e. the union of non-overlapping sets of buyers and sellers,*
2. *the set of connections (edges) between sellers and buyers, which is constructed in a way that for each pair of nodes* $(i, j) \in E_{complete}$*, we independently introduce an edge* (i, j) *in* E_p *with probability* p.

Here is the extension of the algorithm defined in 5 to work in non-complete setting.

Definition 8 (Greedy matching of traders on network).

1. *Initiate the set of trades* T *to* $\emptyset$,
2. *for each edge* $(i^*, j^*) \in E_p$ *calculate the value* $SW(\{(i^*, j^*)\}) = B_{j^*} - S_{j^*}$,
3. *sort the pairs* (i^*, j^*) *of sellers and buyers with respect to* $SW(\{(i^*, j^*)\})$*, in a non-increasing order,*
4. *iterate over sorted pairs* (i^*, j^*)*:*
   (a) *if neither of them has traded, i.e.* $i^* \in \{i : \forall_j (i, j) \notin T\}$ *and* $j^* \in \{j : \forall_i (i, j) \notin T\}$ *as well as* $S_i \leq B_j$ *then make the pair* (i^*, j^*) *trade and update the set:* $T \leftarrow T \cup \{(i^*, j^*)\}$

   until no further trade is possible, i.e. $S_{i^*} > B_{j^*}$.

Since the most time consuming part of the aforementioned algorithm is sorting the $n^2 p$ pairs of sellers and buyers, the computational complexity of the algorithm is $O(n^2 p \log(n^2 p)) = O(n^2 p \log(n))$, where n denotes the total count of both sellers and buyers. Clearly, one may implement a faster algorithm but we do not need it for our experiments.

Greedy algorithm defined in 8 does not guarantee to find a global optimum and maximise the social welfare for a given bipartite graph, unless a graph is complete, as stated before. However, the global optimum can be achieved by

Hungarian algorithm [6] which is much more costly numerically and prohibitive for large bipartite graphs, so it will not be evaluated in this paper. The performance of the algorithm from Definition 8 is reported in Section 4.

Below, we provide a comprehensive summary of various market designs. We focus on four designs in particular, two of which are compared in the results section, and two others that are worth mentioning:

- **Chamberlin's Bilateral Haggling ZIT market** (as defined in Definition 4) serves as our benchmark model. This model is simplistic, assuming no top-down intervention and only bottom-up emergent behaviours of traders. It does not attribute any intelligence to traders in terms of learning, information collection, bargaining capabilities, etc. Consequently, as demonstrated in Section 4 and in the literature [3], this market design suffers from losses in social welfare and market efficiency,
- The **Hungarian Algorithm** is designed to identify market-clearing prices that maximize social welfare within a given bipartite graph, see [5,6,19]. This is achieved by enabling a regulator to dictate the trading partners. Despite its advantages, the algorithm is characterized by high computational complexity of $O(n^3)$.
- **Greedy Matching** (as defined in Definition 8) is our proposed efficient heuristic algorithm. It stands between the two extremes in terms of both computational requirements and resulting market efficiency. Similar to the Hungarian Algorithm, it assumes a market regulator making top-down decisions about who trades with whom. As presented in Section 4, greedy matching exhibits high market efficiency.
- **Double Auction** should be positioned between ZIT's Chamberlin bilateral haggling and our greedy matching algorithm. It allows for some local information exchange, but not to the global extent as in our greedy algorithm. It assumes bottom-up trader behaviour with the institution of public price quoting, facilitating information exchange. High efficiency of this market design for complete bipartite graphs has been demonstrated in [8,20], but there is still no evidence in the literature for sparse bipartite graphs.

4 Simulation Results

This section is dedicated to addressing the research gaps identified and outlined in the research agenda depicted in Figure 1. Specifically, it will showcase exploratory simulation results, with a focus on two fundamental characteristics of markets:

1. market efficiency (Subsection 4.1),
2. trade participation (Subsection 4.2).

Those two metrics will be differentiated based on three main drivers: average degree of trading network, market design and network size. The presented outcomes of market efficiency and trade participation are calculated as the average out of 1000 simulations.

For each simulation, traders are embedded in a newly-generated graph. Since traders are of two types, either seller or buyer, and there is no value from connections within sellers or within buyers as they can make transaction only between (not within) them, it is natural to employ bipartite graph to model interactions between agents, see [6,14]. In this study, random bipartite graph is generated according to the definition 7.

The Julia code employed for this simulation experiment is available at GitHub[4].

4.1 Market efficiency drivers

Figure 2 demonstrates how the market efficiency depends on:

- average degree of bipartite graph, i.e. average number of acquainted traders, with whom an average trader is connected to, $np \in [0.01, 1000]$,
- market design algorithm of decision making, either Zero-Intelligence Traders or greedy matching of traders,
- market size, i.e. the number of sellers and buyers, $n \in \{10, 100, 1000\}$.

In the following subsections we describe the impact of each of these drivers separately based on Figure 2.

Average degree of trading network. Average degree is the expected average number of acquainted traders ($= np$), but it is also a partial measure of network density, i.e. p, the probability of edge existence. Based on Figure 2a, the average degree comes as the major driver of market efficiency. The basic observation is that the larger the average degree, the higher market efficiency for both market design decision mechanisms of ZIT and the greedy matching of traders regardless of the number of traders.

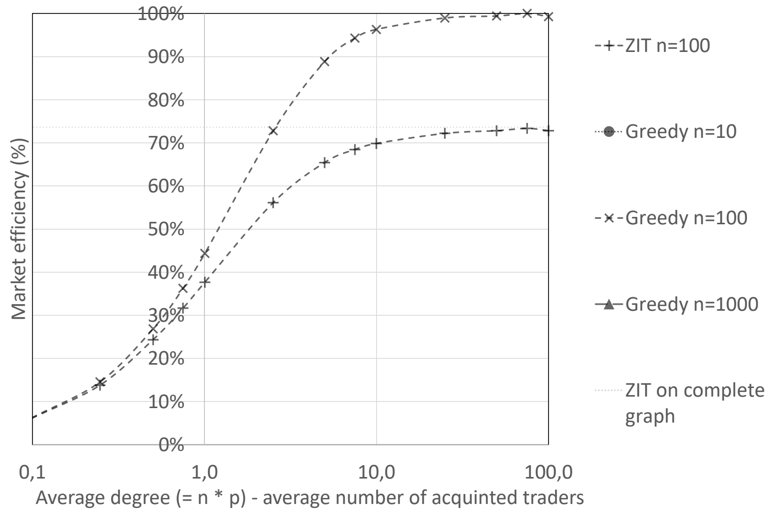
The impact of average degree on market efficiency is not constant, but most significant for degrees between 0.1 and 10, accounting for approximately 90% of the change. This is evident from the steepest ascent of curves in Figure 2b. Degrees less than 0.1 or greater than 10 have minimal effect on market efficiency. An average degree of 5 and 10 achieves roughly 89% and 95% of potential market efficiency[5], respectively. This suggests that moderately dense markets with only 5 or 10 acquainted traders can achieve high potential market efficiency, which has significant implications for policy makers and market designers. From a traders' perspective, they should aim to have at least 5 acquainted traders to achieve roughly 89% of potential profits or consumer surplus.

Market design. Two market designs are considered in Figure 2a:

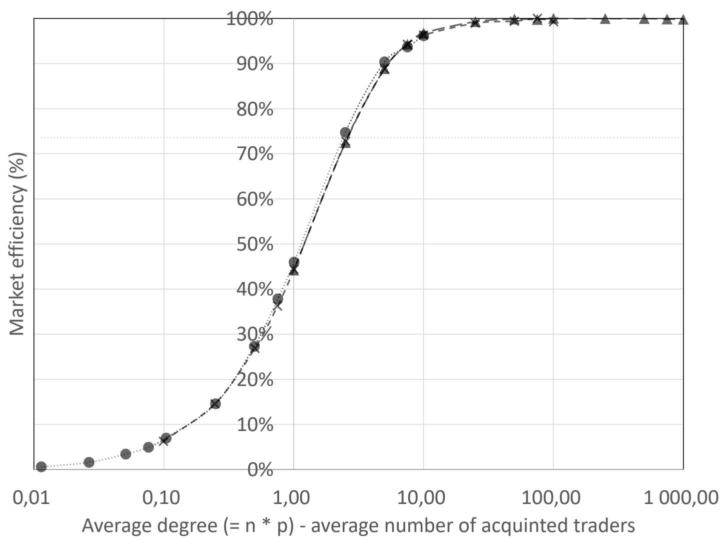
- Zero-Intelligence Traders (ZIT) like in the Chamberlin's higgling market,

[4] `https://github.com/Matzawisza/TradeInNetwork`

[5] Potential market efficiency is the maximum market efficiency in a complete graph for a given market design, i.e., it is roughly 73.6% for ZIT and 100% for a greedy matching of traders.



(a) Superiority of greedy matching over ZIT ($n = 100$)



(b) No market size impact for greedy matching

Fig. 2: Market efficiency comparison between ZIT and greedy matching for varying average degree ($np \in [0.01, 1000]$) and market size ($n \in 10, 100, 1000$)
Source: Own work

– greedy matching of traders, as described in Definition 8.

The choice of market design has the enormous impact on the actual and potential market efficiency. The choice is especially crucial for markets with the average degree of at least 1, so nearly for all markets in practice. The market efficiency of a greedy matching design in comparison to ZIT exhibits following properties:

– greedy matching achieves higher market efficiency than ZIT for all (considered in simulations) values of the average degree and market sizes,
– greedy matching is able to achieve nearly 100% market efficiency, if the trading network is dense enough, while the maximum market efficiency of ZIT is only roughly 73.6% for a complete graph.
– the difference between greedy matching and ZIT starts widening significantly from the average degree of 1 and continues this significant widening till 10 of acquainted traders, when it achieves nearly the maximum difference between two designs of roughly 26.4%.

Based on the above properties we come up with following recommendations for policymakers, which depend on the average number of acquainted traders:

– for the average degree of at least 1, the greedy matching is strictly preferred over ZIT market design, as the difference between the two is of significant magnitude, i.e. at least 5 percentage points. The highest consequence of this market design choice happens for the average degree of at least 10, where the difference achieves nearly its maximum of 26.4%,
– for the average degree lower than 1, even though the greedy matching is still better than ZIT, the difference between both market designs is of less significant magnitude (although relatively it might still matter), so the choice of any particular design is of less importance. In this case, a better policy recommendation would be to increase the average degree of a network.

Market size. Market size denoted by parameter n seems to be a major characteristics of all real-world markets. The intuition goes that the larger market the more trading opportunities and stronger market forces push it to equilibrium state. Therefore, market size is one of tree parameters considered in our simulation and its impact is depicted in Figure 2b.

The small market of 10 sellers and 10 buyers is marginally better performing in terms of market efficiency than larger markets composed of 100 or 1000 sellers and buyers although, the impact is of negligible size, i.e. maximum recorded difference is 3.6% between $n = 10$ and $n = 100$ for average degree of 10. The further increase of market size from 100 and 1000 does not decrease to the same degree the market efficiency (the maximum difference is 0.6% for the average degree of 75).

The difference of market efficiency due to market size is more pronounced although still negligible for:

- ZIT (the maximum within difference of 3.6%) rather than greedy matching (the maximum within difference of 1.9%) market design,
- the average degree between 1 and 10 (the maximum difference within this range is 3.6% for the average degree of 10) rather than outside this range (the maximum difference outside this range is 1.6% for the average degree of 0.8).

4.2 Trade participation drivers

Trade participation, i.e. the total volume of transactions out of all possible transactions, is another metrics of interest to both researchers and policy makers. As discussed in Subsection 3.1, the equilibrium volume is a necessary condition for the market efficiency of 100%. For the parametrization of our simulation, the equilibrium volume is equivalent to the trade participation of 50%. Hence, trading above or below this level will imply no market efficiency. The dependence of trade participation on: (1) the average degree of trading network, (2) market design algorithm of decision making, and (3) market size is depicted in Figure 3 and discussed in following subsections.

Average degree of trading network. Impact of the average degree of trading network on trade participation is positive, as indicated in Figure 3a, although the impact is not constant, but resembles logistic shape relationship.

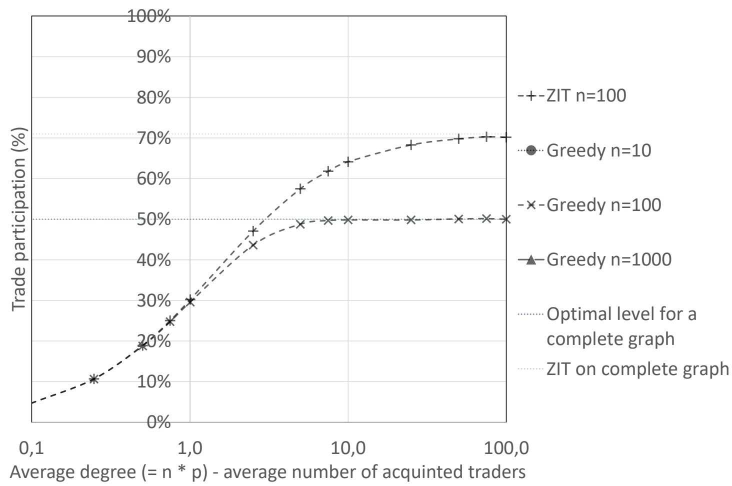
For low values of the average degrees, up to 1, the participation is increasing from roughly 0.5% for the average degree of 0.01 till roughly 30% of trade participation for the average degree of 1 for the market design of either ZIT or greedy matching. The low trade participation is due to the sparse trading network with many traders having no connections to other traders and not being able to make any transactions. Also, those traders who have 1 or a few more connections (but not many) might find themselves unable to make a trade, if neither of possible trading pair can generate a social welfare-improving transaction. Hence, the probability of trading is low.

For medium values of average degrees between 1 and 10, the trade participation is increasing further and achieving completely (for greedy matching) or nearly (for ZIT) its maximum level at average degree of 10. However, this maximum level of participation is different for each market designs, which will be discussed in the next subsection.

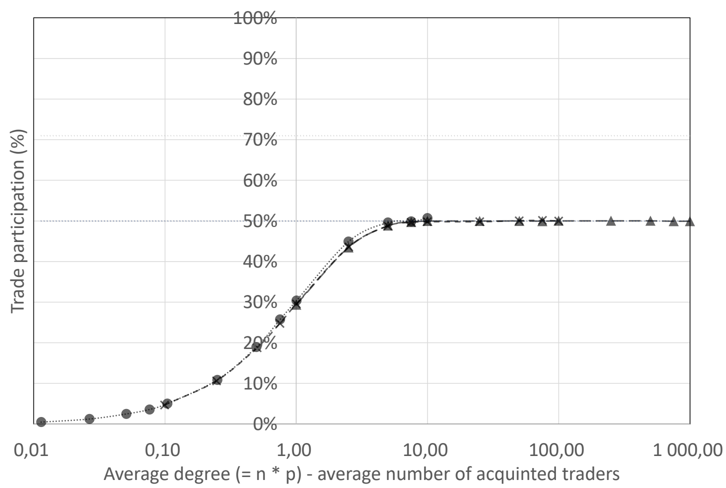
For large values of the average degrees, higher than 10, the ZIT market design still grow to achieve its maximum level of roughly 71%, while greedy matching is stable at the level of 50%.

The average degree can be as low as 2.5% to achieve 87.2% of maximum participation market rate for greedy matching and only 5 acquainted traders for achieving 97.5% of maximum trade participation.

Market design algorithm of decision making. Market design choice is a major driver of the trade participation, especially for the average degree higher



(a) Too much trading of ZIT ($n = 100$)



(b) No market size impact for greedy

Fig. 3: Trade participation comparison between ZIT and greedy matching for varying average degree ($np \in [0.01, 1000]$) and market size ($n \in 10, 100, 1000$) **Source:** Own work

than 10, see Figure 3a. For the average degree lower than 1 both considered market designs do not differ in terms of trading participation. The difference starts widening from the average degree of roughly 1 on and achieves nearly its maximum discrepancy at and above 10 of acquainted traders, which is 21%.

The greedy matching algorithm achieves optimal participation rate of 50% at the average degree of 7.5%. The ZIT design, as we know already, results in "too much trading" and the trade participation of roughly 71% for more dense networks of 100 trading partners for each trader.

Market size. Analogously as for market efficiency, also here the market size is of negligible impact, see Figure 3b. The largest impact of market size is noticed when comparing ZIT designs of 10 and 100 traders. It turns out that smaller market exhibits higher trade participation of 2.2%.

5 Conclusions and Further Research

This paper investigated the effects of market design, market size, and trading network structure on market efficiency and trade participation rate. Two market designs were considered: Zero-Intelligence Traders (ZIT) in Chamberlin's bilateral higgling market and a greedy matching of traders. Both sellers and buyers were embedded in a random bipartite graph with varying network density. Market sizes ranged from 20 to 2000 traders.

Simulations showed that greedy matching outperforms ZIT for non-sparse trading networks. Market efficiency can be significantly improved for both market designs by increasing the average degree of trading networking from 1 to 5 or 10, enabling greedy matching to achieve 89% and 95% of its maximum efficiency, respectively. Contrary to popular belief, market size had no significant impact.

Our model, presented in this paper, offers a simplified representation of traders in a bipartite random graph. However, it is important to acknowledge that real-world scenarios are often more intricate and dynamic. Our model does not incorporate several features observed in real-world markets, such as the qualities of ties among traders, e.g. strong or weak ties, degree distribution of bipartite graphs exhibiting power-law distributions, phenomena such as "rich get richer", communities, assortativity, homophily, aversion, and the dynamic nature of such markets. These elements introduce complexities that our model may not fully capture. Despite these limitations, the authors conjecture—though not explicitly proven or demonstrated—is that the inclusion of these more realistic network features would primarily impact the quantitative results of our model, while leaving the qualitative outcomes relatively stable. This conjecture is based on fact that the qualitative results are usually more robust than quantitative ones. However, it is crucial to interpret these results within the context of these inherent complexities and limitations. Further research is needed to validate this assumption and to quantify the potential impact of these real-world features on our model's outcomes.

Our conducted simulations served as a tool to pinpoint intriguing issues. We are now shifting our focus towards proving theorems about the processes that were simulated. Looking ahead, our research will aim to evaluate the possibility of obtaining analytical results for some manageable models of network, market design, and decision process, offering a contrast to simulation results. This future work will provide a more rigorous understanding of the systems under study. Besides that, the conducted study did not fully exploit the research agenda outlined in Figure 1. Future investigations could include:

- Enriching the network structure beyond a random bipartite graph to exhibit more realistic features of real-world networks, such as "rich get richer" [1], "birds of a feather flock together", "six degrees of separation", and small-world property [23] or with community structure [18,13],
- Evaluating the impact of network structure on Double Auction design.

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