CHASING ROBBERS ON RANDOM GRAPHS: ZIGZAG THEOREM

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ABSTRACT. In this paper, we study the vertex pursuit game of *Cops and Robbers* where cops try to capture a robber on the vertices of the graph. The minimum number of cops required to win on a given graph G is the cop number of G. We present asymptotic results for the game of Cops and Robber played on a random graph $G(n,p)$ for a wide range of $p = p(n)$. It has been shown that the cop number as a function of an average degree forms an intriguing zigzag shape.

1. Introduction

The game of *Cops and Robbers*, introduced independently by Nowakowski and Winkler [11] and Quilliot [13] over twenty years ago, is played on a fixed graph G, and is our focus in this paper. We will always assume that G is undirected, simple, and finite. There are two players, a set of k *cops*, where $k \geq 1$ is a fixed integer, and the *robber*. The cops begin the game by occupying any set of k vertices (in fact, for a connected G, their initial position does not matter). The robber then chooses a vertex, and the cops and robber move in alternate rounds. The players use edges to move from vertex to vertex. More than one cop is allowed to occupy a vertex, and the players may remain on their current positions. The players know each others current locations. The cops win and the game ends if at least one of the cops eventually occupies the same vertex as the robber; otherwise, that is, if the robber can avoid this indefinitely, he wins. As placing a cop on each vertex guarantees that the cops win, we may define the *cop number*, written $c(G)$, which is the minimum number of cops needed to win on G. The cop number was introduced by Aigner and Fromme [1] who proved (among other things) that if G is planar, then $c(G) \leq 3$. For more results on vertex pursuit games such as *Cops and Robbers*, the reader is directed to the surveys on the subject [2, 8, 9]. Here we mention only that the most important open problem in this area is the Meyniel's conjecture (communicated by Frankl [7]). It states that $c(n) = O(\sqrt{n})$, where $c(n)$ is the maximum of $c(G)$ over all n-vertex connected graphs. If true, the estimate is best possible as one can construct a bipartite graph based on the finite projective plane with the cop number of order at least $\sqrt{n}$. Up until recently, the best known upper bound of $O(n \log\log n/\log n)$ was given in [7]. It took 20 years to show that $c(n) = O(n/\log n)$ proved in [6]. Today we know that the cop number is at most $n2^{-(1+o(1))\sqrt{\log n}}$ (which is

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still $n^{1-o(1)}$) for any connected graph on n vertices (the result obtained independently by Lu, Peng [10] and Scott, Sudakov [14]).

If one looks for counterexamples for Meyniel's conjecture it is natural to study first the cop number of random graphs. Let us recall that the binomial random graph $G(n,p)$ is defined as a random graph with vertex set $[n]=\{1,2,\dots,n\}$ in which a pair of vertices appears as an edge with probability p, independently for each such a pair. As typical in random graph theory, we shall consider only asymptotic properties of $G(n,p)$ as $n\to\infty$, where $p=p(n)$ may and usually does depend on n. Consequently, all inequalities are assumed to hold only for n large enough. We say that an event in a probability space holds *asymptotically almost surely* (*a.a.s.*) if its probability tends to one as n goes to infinity.

Let us first describe briefly some known results on the cop number of $G(n,p)$. Bonato, Wang, and the second author of this paper started investigating such games in $G(n,p)$ random graphs and their generalizations used to model complex networks with a power-law degree distribution (see [4, 5]). From their results it follows that if $np=n^{\alpha+o(1)}$, where $1/2<\alpha\le 1$, then a.a.s.

$$c(G(n,p))=\Theta(\log n/p)=n^{1-\alpha+o(1)} \tag{1}$$

and $c(G(n,n^{-1/2+o(1)}))=n^{1/2+o(1)}$ a.a.s. In fact, for constant p we get much better concentration, namely, it has been shown that

$$c(G(n,p))=(1+o(1))\log_{\frac{1}{1-p}} n.$$

In order to get a constant cop number that does not grow with the size of the graph, $p=p(n)$ must tend to one as n goes to infinity (see [12] for more).

On the other hand, Bollobás, Kun, and Leader [3] showed that the cop number of $G(n,p)$ is always bounded from above by $n^{1/2+o(1)}$ and this bound is achieved at the other end of the spectrum, that is, for sparse random graphs. More precisely, they showed that whenever $p(n)\ge 2.1\log n/n$, then a.a.s.

$$\frac{1}{(np)^2}n^{\frac{1}{2}\frac{\log\log(np)-9}{\log\log(np)}}\le c(G(n,p))\le 160000\sqrt{n}\log n\,. \tag{2}$$

Hence, if we ignore a logarithmic factor, $G(n,p)$ cannot be used to construct a counterexample for the Meyniel's conjecture.

Since if either $np=n^{o(1)}$ or $np=n^{1/2+o(1)}$, then a.a.s. $c(G(n,p))=n^{1/2+o(1)}$, it would be natural to assume that the cop number of $G(n,p)$ is close to $\sqrt{n}$ also for $np=n^{\alpha+o(1)}$, where $0<\alpha<1/2$. Note that for this range of p, the result presented in (2) implies only that

$$n^{1/2-2\alpha+o(1)}\le c(G(n,p))\le n^{1/2+o(1)},$$

which is not tight and gives no lower bound for $1/4<\alpha<1/2$. We show that the actual behaviour of $c(G(n,p)$ is more complicated. Let function $f:(0,1)\to\mathbb{R}$ be defined as

$$f(x)=\frac{\log\bar{c}(G(n,n^{x-1}))}{\log n}\,,$$

where $\bar{c}(G(n,p))$ denotes the median of the cop number for $G(n,p)$, has a characteristic zigzag shape (see Figure 1). We are going to show that f has the following form

$$f(x) = \begin{cases} \alpha j, & \text{if } \frac{1}{2j+1} \le \alpha \le \frac{1}{2j} \text{ for some } j \ge 1; \\ 1 - \alpha j, & \text{if } \frac{1}{2j} < \alpha < \frac{1}{2j-1} \text{ for some } j \ge 1. \end{cases}$$

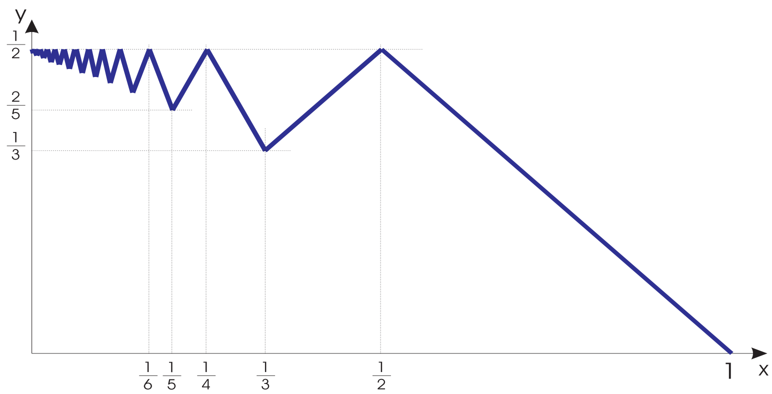

FIGURE 1. The 'zigzag' function f.

Our main result is as follows.

Theorem 1.1. *Let $0 < \alpha < 1$ and $d = d(n) = np = n^{\alpha+o(1)}$.*

(i) *If $\frac{1}{2j+1} < \alpha < \frac{1}{2j}$ for some $j \ge 1$, then a.a.s.*

$$c(G(n,p)) = \Theta(d^j)\,.$$

(ii) *If $\frac{1}{2j} < \alpha < \frac{1}{2j-1}$ for some $j \ge 1$, then a.a.s.*

$$\Omega\left(\frac{n}{d^j}\right) = c(G(n,p)) = O\left(\frac{n}{d^j}\log n\right)\,.$$

Note that in the above result we skip the case when $np = n^{1/k+o(1)}$, for some natural k. We have done it for technical reasons: our argument for the lower bound for $c(G(n,p))$ uses Corollary 2.6 from [15] which is stated only for $np = n^{\alpha+o(1)}$, where $\alpha \ne 1/k$. Clearly, one can repeat the argument given in [15], which is a very nice but slightly technical application of the polynomial concentration method inequality by Kim and Vu. However, in order to make paper easier and more compact we have decided to apply ready-to-use Vu's result and concentrate on the 'linear' parts of the graph of the zigzag function. Nonetheless, one can expect that, up to a factor of $\log^{O(1)} n$, our result extends naturally also to the case $np = n^{1/k+o(1)}$ as well.

2. PROOF OF THE MAIN RESULT

Let us start with the following result on typical properties of $G(n,p)$. Let $N_i(v)$ denote a set of vertices that are within distance at most i from v.

Lemma 2.1. *Let $0 < \varepsilon < 0.1$, $\varepsilon < \alpha < 1 - \varepsilon$, and $d = d(n) = np = n^{\alpha + o(1)}$. Then a.a.s. for every vertex v of $G(n,p)$ the following holds.*

(i) *For every $1 \le i \le 1/\alpha$ such that $d^i \le \varepsilon^2 n$ we have*

$$(1-\varepsilon)d^i \le |N_i(v)| \le (1+\varepsilon)d^i \ ,$$

Furthermore, if $k = 1/\alpha \in \mathbb{N}$ and $d^k \ge \varepsilon^2 n$, then

$$|N_k(v)| \ge 0.9\varepsilon^2 n \ .$$

(ii) *For every $1 \le i \le 1/(2\alpha)$ and vertices $v_1, v_2, \dots, v_k$ we have*

$$\left| \bigcup_{j=1}^{k} N_{i+1}(v_j) \right| \ge 0.5 \min\{k(0.1d)^{i+1}, n\}. \tag{3}$$

(iii) *If $w \in N_i(v)$ for some i, $2 \le i < 1/\alpha$, then v and w are joined by fewer than $2/(1 - i\alpha)$ paths of length i.*

Furthermore, if $\ell = \lfloor 1/\alpha \rfloor + 1 < 1/\alpha + 1$, then the number of paths of length ℓ joining v and w are bounded from above by $\frac{3}{1-(\ell-1)\alpha} \frac{d^\ell}{n}$.

(iv) *If $i < 1/\alpha$, then each edge of $G(n,p)$ is contained in at most εd cycles of length at most $i+2$.*

Proof. The estimates in (i) for the size of neighbourhoods of vertices are well-known and follow easily from Chernoff's inequality.

In order to show (ii) let us choose k vertices and generate their $(i+1)$th neighbourhoods sequentially, each time disregarding vertices which belong to vertices whose neighbourhoods have been already found. Then, by Chernoff's bounds, the probability that for a vertex v we found fewer than $(0.1d)^{i+1}$ new vertices in its $(i+1)$th neighbourhood, provided we have used no more than $n/2$ vertices so far, is smaller than n^{-4}. If (3) does not hold, then the number of used vertices is smaller than $n/2$ and at least half among k vertices must have fewer than $(0.1d)^{i+1}$ vertices in its $(i+1)$th neighbourhood. However, probability of such an event is bounded from above by

$$\sum_{k=1}^{n} \binom{n}{k} 2^k (n^{-4})^{k/2} \le \sum_{k=1}^{n} (4n^{-2})^{k/2} = O(n^{-1}).$$

The first part of (iii) can be easily verified using the first moment method; the second part of (iii) is a consequence of (i) (for $i = \lfloor 1/\alpha \rfloor$), the first part of (iii) and Chernoff's bounds.

In order to verify (iv) it is enough to check that for any given pair of vertices r_1, r_2, and j such that $2 \le j \le i+1$, the probability that $G(n,p)$ contains more than $\varepsilon d/(2i)$ (r_1, r_2)-paths of length j is $o(n^{-2})$. Denote the number of such paths by $X_j^{r_1,r_2}(n,p)$. Then, for the expectation of $X_j^{r_1,r_2}(n,p)$ we get

$$\mathbb{E}X_j^{r_1,r_2}(n,p) = \binom{n-2}{j-1}(j-1)!p^j < 2\frac{d^j}{n} \le 2d\frac{d^i}{n} \le \frac{\varepsilon d}{4i} \, .$$

Now, choose $p' > p$ in such a way that

$$\mathbb{E}X_j^{r_1,r_2}(n,p') = \frac{\varepsilon d}{4i} \ge \log^{2(j+1)} n \, .$$

Then, by a result of Vu (see [15], Corollary 2.6) it follows that for some constant $a > 0$,

$$\begin{aligned}\Pr(X_j^{r_1,r_2}(n,p') > \varepsilon d/(2i)) &\leq \Pr(X_j^{r_1,r_2}(n,p') > 2\mathbb{E}X_j^{r_1,r_2}(n,p'))\\ &\leq \exp\big(-a(\mathbb{E}X_j^{r_1,r_2}(n,p'))^{1/(j+1)}\big)\\ &\leq \exp\big(-a\log^2 n\big) = o(n^{-2})\,.\end{aligned}$$

Consequently,

$$\Pr\left(X_j^{r_1,r_2}(n,p) > \frac{\varepsilon d}{2i}\right) \leq \Pr\left(X_j^{r_1,r_2}(n,p') > \frac{\varepsilon d}{2i}\right) = o(n^{-2})\,,$$

and the assertion follows. □

The upper bound for $c(G(n,p))$ follows from the following result.

Lemma 2.2. *Let $j \geq 1$ and $d = d(n) = pn$.*

(i) *Let $n^{1/(2j+1)} \leq d \leq n^{1/(2j)}$ and $\gamma = \lceil n \log n / d^{2j+1} \rceil$. Then a.a.s.*

$$c(G(n,p)) = O\big(d^j \gamma\big)\,.$$

(ii) *If $n^{1/(2j+2)} \leq d \leq n^{1/(2j+1)}$, then a.a.s.*

$$c(G(n,p)) = O\Big(\frac{n}{d^{j+1}} \log n\Big)\,.$$

Proof. Assume first that $n^{1/(2j+1)} \leq d \leq n^{1/(2j)}$. We describe an 'immediate pursuit' strategy for cops and then prove that a.a.s. it is winning in the game. We place $\beta n = 5000(10d)^j \gamma$ cops uniformly at random on vertices of $G(n,p)$. Then, the robber selects his vertex v. Now, we assign to each vertex u in $N_j(v) \setminus N_{j-1}(v)$ the unique cop that occupies a vertex in $N_{j+1}(u)$. If this can be done, then cops assigned to vertices are moving into their destinations and after $j+1$ steps the robber is surrounded. Finally, the cops move towards the robber eventually capturing him.

In order to show that the above strategy is a.a.s. winning, we use Hall's theorem for matchings in bipartite graphs. Thus, let us fix any vertex v and $S \subseteq N_j(v) \setminus N_{j-1}(v)$ with $|S| = k$. Let

$$k_0 = \max\{k : (0.1d)^{j+1} k < n\}.$$

It follows from Lemma 2.1(ii) that if $k \leq k_0$, then the number of cops that occupy $\bigcup_{u\in S} N_{j+1}(u)$ is bounded from below by the Bernoulli random variable $B(M,\beta)$, where $M \geq 0.1k(0.1d)^{j+1}$. The expectation of this random variable is $M\beta \geq 50k \log n$, so, using the Chernoff's bounds, we infer that the probability that there are fewer than k cops in $\bigcup_{u\in S} N_{j+1}(u)$ is less than $\exp(-4k\log n)$. Since

$$\sum_{k=1}^{k_0} \binom{|N_j(v)|}{k} \exp(-4k\log n) \leq \sum_{k=1}^{n} n^k \exp(-4k\log n) = O(n^{-2}),$$

with probability $1 - O(n^{-2})$ the necessary condition in the statement of the Hall's theorem holds for all sets of cardinality at most k_0.

In a similar way, again by Lemma 2.1(ii), if $k_0 \le k \le |N_j(v)| \le 2d^j$, then the Chernoff's bound implies that the number of cops in $\bigcup_{u \in S} N_{j+1}(u)$ is at least $\frac{1}{4} n\beta \ge 50d^j > |N_j(v)|$ with probability at least $1 - \exp(-4d^j)$. Since

$$\sum_{k=k_0+1}^{|N_j(v)|} \binom{|N_j(v)|}{k} \exp(-4d^j) \le 2d^j 2^{2d^j} \exp(-4d^j) = O(n^{-2}),$$

the Hall's necessary condition holds with probability $1 - O(n^{-2})$ provided the robber starts from vertex v. Since the robber has n vertices available to start from, we have shown that a.a.s. the robber will be surrounded after his first j moves. It is also easy to show that a.a.s. there is always a matching between $N_{i+1}(v) \setminus N_i(v)$ and $N_i(v) \setminus N_{i-1}(v)$ saturating each vertex in a smaller set, for all $i = 1, 2, \dots, j$; that is, a.a.s. the cops can move toward the robber tightening the loop, and win the game in the next j moves.

Note that if $(n \log n)^{1/(2j+1)} \le d \le n^{1/(2j)}$, then $\gamma = 1$; when d is approaching $n^{1/(2j+1)}$ the constant γ grows, and becomes $\log n$ for $d = n^{1/(2j+1)}$. The reason for introducing an additional factor γ in the proof follows from the fact that in order to use Chernoff's bound for small k's, the expected number of cops in the $(j+1)$th neighbourhood of a vertex from $N_j(v)$ must be $\Omega(\log n)$. If $(n \log n)^{1/(2j+1)} \le d \le n^{1/(2j)}$, and we place $\Theta(d^j)$ cops in the graph, then the expected number of cops in $N_{j+1}(w)$ is $\Theta(d^{j+1} d^j / n) = \Theta(d^{2j+1}/n) = \Omega(\log n)$, but when $n^{1/(2j+1)} \le d \le (n \log n)^{1/(2j+1)}$ the extra factor γ is needed.

One can mimic the above argument to show that if $n^{1/(2j+2)} \le d \le n^{1/(2j+1)}$, then a.a.s. $\beta n = 5000 n \log n / (0.1d)^{j+1}$ cops can win the game. The difference in the estimates of the cop number follows from the fact that in the immediate pursuit strategy only cops who are within distance $2j+1$ from the robber are 'active', that is, they can take part in the chase. In the previous case all but a small fraction of cops were active. Now in our team we have

$$\Theta\big(\beta d^{2j+1}\big) = \Theta\big(d^j \log n\big)$$

active cops only. Thus, as the $(j+1)$th neighbourhoods of vertices from the jth neighbourhood of v are nearly disjoint, the expected number of cops in the neighbourhood of $w \in N_j(v)$ is $\log n$, as in the previous case. This little adjustment seems to be only a cosmetic technical difference but it plays an important role and has a big impact on the main result implying the zigzag shape of the function $f(x)$ we study. □

The lower bound for $c(G(n,p))$ is given in the two following results.

Lemma 2.3. *Let* $\frac{1}{2j+1} < \alpha < \frac{1}{2j}$ *for some natural* $j \ge 1$, $c = c(j, \alpha) = \frac{3}{1-2j\alpha}$ *and* $d = d(n) = np = n^{\alpha + o(1)}$. *Then a.a.s.*

$$c(G(n,p)) \ge \left[\frac{d}{3cj}\right]^j.$$

Proof. Since our result holds a.a.s., without loss of generality, we can assume that the graph we play on satisfies the properties stated in Lemma 2.1. Let us assume that we chase the robber using fewer than $\left[\frac{d}{3cj}\right]^j$ cops. For vertices $x_1, \dots, x_s$ let $\mathrm{C}_i^{x_1,\dots,x_s}(v)$ denote the number of cops in the ith neighbourhood of v in the graph $G(n,p) \setminus \{x_1, \dots, x_s\}$;

in particular, if $v \notin \{x_1, \dots, x_s\}$ then $\mathrm{C}_0^{x_1,\dots,x_s}(v) = 0$ if and only if a vertex v is not occupied by a cop. Before the robber's move, we say that a vertex v occupied by the robber is safe, if for some neighbour x of v we have $\mathrm{C}_0^x(v) = 0$, and

$$\mathrm{C}_{2i-1}^x(v), \mathrm{C}_{2i}^x(v) \le \left[\frac{d}{3cj}\right]^i$$

for every $i = 1, 2, \dots, j$ (such a vertex x will be called a deadly neighbour of v). Note that, since a.a.s. $G(n,p)$ is connected, without loss of generality one can assume that at the beginning of the game all cops are placed at the same vertex w. Then, the robber may choose a vertex v which is at a distance $2j+1$ from w (see Lemma 2.1(i) for $i = 2j$) and so, even if all cops will move to $N_{2j}(v)$, after this move v will remain safe. Hence, in order to prove the lemma, it is enough to show that if v is safe, the robber can move to an unoccupied neighbour y so that y will remain safe after the following move of cops.

We say that for some $r \ge 0$ a neighbour y of v is r-*dangerous* if

(i) $\mathrm{C}_r^{v,x}(y) > 0$ if $r = 0, 1$;

(ii) $\mathrm{C}_r^{v,x}(y) > \left[\frac{d}{3cj}\right]^i$ if either $r = 2i$ or $r = 2i+1$,

where x is a deadly neighbour of v. We shall check now that for every r the number $\mathrm{dang}(r)$ of r-dangerous neighbours of v is smaller than $d/3j$. Since by the assumption that v is safe we know that $\mathrm{C}_s^x(v) \le \frac{d}{3cj}$ for $s \in \{1, 2\}$, the claim follows for $r = 0$ and $r = 1$. In a similar way, from the fact that for a cop occupying a vertex w at a distance i from v, it follows that fewer than c neighbours of v are at a distance $i-1$ from w. Thus, for $r = 2i$, we have

$$\left[\frac{d}{3cj}\right]^{i-1} \mathrm{dang}(2i-1) \le c \cdot \mathrm{C}_{2i}^x(v) \le c\left[\frac{d}{3cj}\right]^i,$$

and consequently

$$\mathrm{dang}(2i-1) \le \frac{d}{3j}.$$

If $r = 2i+1$, then

$$\left[\frac{d}{3cj}\right]^{i} \mathrm{dang}(2i) \le c \cdot \mathrm{C}_{2i+2}^x(v) \le c\left[\frac{d}{3cj}\right]^{i+1},$$

and again

$$\mathrm{dang}(2i-1) \le \frac{d}{3j}.$$

Thus, at most $2d/3$ of neighbours of v are r-dangerous for some $r = 0, 1, \dots, 2j-1$. Now we may use Lemma 2.1(i) and (iv) to infer that there is a neighbour y of v which is not r-dangerous for all $r = 0, 1, \dots, 2j-1$ and x does not belong to the $(2j-1)$-neighbourhood of y in $G(n,p) \setminus \{v\}$. We move the robber to y.

Now it is time for cops to make their move. Because of our choice of the vertex y, we can assure that the upper bound for $\mathrm{C}_r^v(y)$ required for y to be safe will be held for $r = 0, 1, \dots, 2j-1$. Indeed, the best that the cops can do is to decrease the distance between them and the robber by one, but, even if all cops are able to decrease their distance to the robber, the condition that is required holds. Note that we cannot control the number of cops in $N_{2j-1}(y)$ and $N_{2j}(y)$ but both $\mathrm{C}^v_{2j-1}(y)$ and $\mathrm{C}^v_{2j}(y)$ are, clearly, bounded from above by the total number of cops. Thus, after their move y is safe and the assertion follows. $\square$

Lemma 2.4. *Let* $\frac{1}{2j} < \alpha < \frac{1}{2j-1}$ *for some natural number* $j \geq 2$, $\bar{c} = \bar{c}(\alpha) = \frac{3}{1-(2j-1)\alpha}$ *and* $d = d(n) = np = n^{\alpha+o(1)}$. *Then a.a.s.*

$$c(G(n,p)) \geq \left[\frac{d}{3\bar{c}j}\right]^j \frac{n}{\bar{c}d^{2j}}.$$

Proof. The proof is very similar to that of Lemma 2.3. The only difference is that the vertices in the $2j$th neighbourhood can $(2j-1)$-dominate as many as $\bar{c}d^{2j}/n$ neighbours of the vertex occupied by the robber (see Lemma 2.1(iii)). Thus, one needs to modify the definition of a safe vertex, and call a vertex v safe if for some neighbour x of v we have $\mathrm{C}^x_0(v) = 0$, and

$$\mathrm{C}^x_{2i-1}(v), \mathrm{C}^x_{2i}(v) \leq \left[\frac{d}{3\bar{c}j}\right]^i \frac{n^{1-2j\alpha}}{\bar{c}},$$

for every $i = 1, 2, \ldots, j$. Besides this modification the argument remains basically the same. $\square$

Proof of Theorem 1.1. Theorem 1.1 is a straightforward consequence of (1), Lemmas 2.2, 2.3, and 2.4. $\square$

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